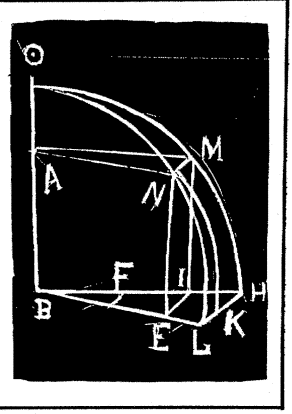
273. Proposed by **A. H. HOLMES**, Brunswick, Maine.

Required a purely geometrical solution of the problem, to find the contents of a solid generated by the revolution of a semi-segment of a circle about the sine of its arc.

Solution by the PROPOSER.

HBO is a quadrant whose revolution about *BO* as an axis generates a hemisphere. *BAF* is a semi-segment of radius$=BO$. Draw *AM* parallel to *HB* and *MI* parallel to *BO*. Suppose the quadrant to revolve about its axis a very small distance, the point *H* moving to *L* so as to generate *HBOLB*, *M* falling on *N*. Through *NE* pass a plane parallel to *HBO*. The semi-segment $HMI=AFB$ generates *HIELMN*; of which the part $EKLN$=part generated by *BAF*.



It is obvious that the volume generated by the semi-segment *BFA* in an entire revolution will equal that generated by *HMI minus* the sum of the solids *HIKEMN* lying about the circumference of the base of the hemisphere.

But $MNHIEK=IE\times$area of the semi-segment and the entire sum of all these is equal to the circumference described by *BI* as radius into the same area. If we put $BO=r$, $BE=c$, $BA=s$, and arc $AF=a$, we obtain for the solid generated by *BOMI*, $\frac{2\pi}{3}(sc^2+r^3-r^2s)$. Consequently, for the solid generated by *HIM*, $\frac{2\pi}{3}(r^2s-sc^2)$. The sum of all the solids $HKEINM$=semi-segment $MHI\times 2\pi c$ $=\pi(car-sc^2)$. Consequently the volume sought is

$$=\frac{2\pi}{3}(r^2s-sc^2)-\pi(car-sc^2)=\pi\left(\frac{sc^2}{3}+\frac{2r^2s}{3}-car\right).$$

Putting $c^2=r^2-s^2$, this becomes $\pi(sr^2-s^3/3-car)$.

GROUP THEORY.

14. Proposed by **O. E. GLENN**, Springfield, Mo.

Hölder has proved* that any group (G) of order $\sum_{i=1}^{n} p_i$ (p_i a prime$\neq p_j$) may be generated as follows: $M^\mu=N^\nu=1$, $N^{-1}MN=M^\alpha$, where $\{M\}$ is the product of all the invariant subgroups of G of prime order and $\{N\}$ is any one of a set of conjugate cyclical subgroups of order ν, ($\sum_{i=1}^{n} p_i=\mu\nu$). Find the generating relations of G in terms of operations of prime order, and express M and N in terms of these operations, for $n=4$.

*See Burnside, *Theory of Groups*, p. 353.